

# TEXTO PARA DISCUSSÃO

The Harris-Todaro Labor Allocation  
Mechanism as an Evolutionary Game

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Nº 03/2004



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## The Harris-Todaro Labor Allocation Mechanism as an Evolutionary Game

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### Abstract

The Harris-Todaro model is reinterpreted under an evolutionary game approach, including the behavior of migrants under bounded rationality in a process of imitation or learning. A first feature is an improvement upon Harris-Todaro by replacing the sign-preserving function of the adjustment mechanism by a replicator dynamics, which is deduced from the assumptions about the interaction process among heterogeneous workers in a bounded rationality context. A second feature is an interpretation of the equilibrium condition as a mixed strategy equilibrium of an evolutionary game. The Harris-Todaro equilibrium condition then appears as a spontaneous result of the model.

**Key words:** migration, labor market, evolutionary game, bounded rationality

**JEL codes:** J6, O15, C7

Developing countries generally show some kind of dualism in their labor markets, be it with respect to production structure (traditional and modern), to geographical location (rural and urban), to legal nature of the activities (formal and underground), or to the composition of the labor force (skilled and non-skilled). As shown by Agénor and Montiel (1996, p. 63), duality in the labor market implies a segmentation, i.e., a situation in which identical workers earn different wages depending on where they are employed.

Todaro (1969) has built up a seminal model to analyze rural-urban migration in a developing country, extending and formalizing ideas from various authors that followed Lewis (1954). As the main assumption of the model, the decision of the rural worker to migrate depends upon expected differential wage. In Michael Todaro's analysis, the decision to move is seen as an investment decision tied to expected net returns. These expected returns crucially depend upon the probability of getting a job in the modern, or formal, sector versus a job at the traditional urban sector, also called underground or informal. The model takes this probability as endogenous and influenced by the creation of urban employment and the number of unemployed urban workers. The objective of the model is to show how the

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employment rate tends to an equilibrium below full employment even in the long run. Harris and Todaro (1970) is a general equilibrium analysis in which the artificial upholding of the wage differential between the rural and urban sectors leads to an inefficient equilibrium. Although the informal sector is now excluded from the model, the concept of expected wage is kept. What determines the expected wage is employment or unemployment in the urban sector.

Despite defending the principle that workers take into consideration the present value of the expected real income flow, Todaro (1969) as well as Harris and Todaro (1970) in fact work with the postulate of myopic expectations. Todaro (1969, sect. IV) formally obtains his main conclusions from the assumption that the time horizon of the workers is only one period. Thus, in their model, the differential in real income for time  $t$  appears in place of the differential of the present value of the flow of expected real income along two or more periods. This is done in the adjustment mechanism associated with the aggregate supply of workers to the urban sector. Todaro (1969, p. 143, n. 10) justifies this by saying that this "... assumption is made necessary by mathematical convenience but is *in fact probably a more realistic formulation in terms of actual decision making in less developed nations ...*" [Our italics.] We consider this implicit idea of bounded rationality of the workers, and propose a model based on the tools of the theory of evolutionary games.<sup>1</sup>

In our model, the migratory movement of workers is interpreted as a process of imitation/learning in an environment of bounded rationality.<sup>2</sup> With the help of an evolutionary game, we deduce a replicator dynamics that replaces the adjustment mechanism *postulated* both in Todaro (1969) and Harris and Todaro (1970). Starting with their same general equilibrium structure, but including it in our model of evolutionary game, we show that their classical results appear as a consequence of the interaction among economic agents that try to get higher wages in a context of bounded rationality.

## **I. The Harris-Todaro Analysis of the Rural-Urban Migratory Movement**

Harris and Todaro (1970) study the migration of workers in a two-sector economic system, called *rural* and *urban*. These sectors differ by the kind of goods they produce and by

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<sup>1</sup> On this theory, see Weibull (1995), Vega-Redondo (1996), Samuelson (1997), and Hofbauer and Sigmund (1998).

<sup>2</sup> The model is based on one initially presented in Silveira (2001), where it was used to represent the competition among workers along business cycles.

the technology of production. The rural sector specializes in the production of an agricultural good whose productive process is described by the following production function:<sup>3</sup>

$$(1) \quad x_a = g(n_a), \quad \text{with } g'(n_a) > 0 \text{ and } g''(n_a) < 0,$$

in which  $x_a$  is the production level of the agricultural good and  $n_a$  is the input quantity of workers (in units of population) to the rural sector. The endowment of land and the stock of capital of this sector are given for the period of analysis.

Similarly, the urban sector has the following production function:

$$(2) \quad x_m = f(n_m), \quad \text{with } f'(n_m) > 0 \text{ and } f''(n_m) < 0,$$

where  $x_m$  is the production level of the manufactured good and  $n_m$  is the quantity of workers (in units of population) used in the production of manufactured goods. The stock of capital of this sector is also given during the period of analysis.

Both goods and labor markets are perfectly competitive. However, there is segmentation in the labor market. For the rural sector, the real wage, perfectly flexible, is equal to the marginal productivity of labor:

$$(3) \quad \omega_a = g'(n_a)p,$$

where  $\omega_a$  is the real wage and  $p$  is the price of the agricultural good, both expressed in terms of the manufactured good.<sup>4</sup>

For the urban sector, besides perfectly competitive markets, it is assumed a minimum wage,  $\bar{\omega}_m$ , fixed at a level above equilibrium in this labor market. Formally:

$$(4) \quad \bar{\omega}_m = f'(n_m^*) \text{ such that } n_m^* \leq n_u,$$

where  $n_u$  is the quantity of workers (in population units) in the urban sector.

As to the terms of trade between the two sectors, it is assumed that the relative price varies according to the relative scarcity between them, i.e.,

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<sup>3</sup> Our notation is slightly different from the original one of Harris and Todaro (1970). We work, without loss of generality, with the number of workers normalized in units of total population, since this is the adequate variable for the evolutionary game that we will develop in the next section.

<sup>4</sup> Thus the manufactured good is taken as the numeraire.

$$(5) \quad p = \rho(x_m/x_a), \text{ with } \rho'(x_m/x_a) > 0.$$

Within the period of analysis, the total population of workers in the economy is given at a level  $\bar{n} > 0$ . Without loss of generality, we may normalize the population size to  $\bar{n} = 1$ . Since by assumption there are only two sectors and rural prices are wholly flexible,<sup>5</sup> then at any time the following equality is verified:

$$(6) \quad n_a + n_u = 1.$$

With these assumptions, there is a general equilibrium structure that determines, for a given urban population of workers, a relative price structure,  $(\omega_a, p)$ , a sector allocation of labor,  $(n_a, n_m)$ , and the sector production,  $(x_a, x_m)$ . The urban sector minimum wage being fixed at  $\bar{\omega}_m$ , the firms of this sector, as price takers, maximize profits by employing  $n_m^*$  and producing  $x_m^*$ . In other words, given  $\bar{\omega}_m$ , by (4) we get  $n_m^*$ ; then we get  $x_m^*$  with the help of (2). Therefore, during the period of analysis, given the minimum wage, the employment level and production of the urban sector are unaltered.

Once the distribution of workers between the sectors of the economy is given, the real wage of the rural sector and the terms of trade between the sectors adjust until all the rural workers are employed and the full employment production level is reached. In other words, given  $n_u$ , we get  $n_a$  by (6). With  $n_a$  and (1), we obtain the full employment production of the rural sector,  $x_a$ . The sector production levels determine the terms of trade  $p$  through equation (5). Finally, the real wage of the rural sector in terms of the manufactured good,  $\omega_a$ , is obtained by (3), given  $n_a$  and  $p$ . In short, the vector  $(\omega_a, p, n_a, n_m^*, x_a, x_m^*)$  configures a temporary equilibrium or a shortrun, which is defined for a given vector of exogenous variables<sup>6</sup> and a given urban population of workers  $n_u$ . This temporary equilibrium will be disturbed when a redistribution of the population of workers between sectors happens, i.e., when there is a migration of workers that changes the urban population.

Harris and Todaro, for determining the long run equilibrium, argue that the rural workers reckon, in their decision on migrating to the urban area, the expected wage,  $\omega_u^e$ , defined as:

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<sup>5</sup> This means that there is full employment in the rural area.

<sup>6</sup> The endowment of land, the stocks of capital and the minimum wage.

$$(7) \quad \omega_u^e = \frac{n_m^*}{n_u} \bar{\omega}_m.$$

The key assumption of the model of Harris and Todaro is that there will be a migratory flow from the rural to the urban sector while the expected urban real wage is higher than in the rural sector. Thus, the *long run equilibrium*, i.e., the absence of a rural-urban migratory flow is established when the urban worker population reaches a level such that the expected urban real wage equates the rural real wage, i.e.:

$$(8) \quad \omega_u^e = \omega_a.$$

This equality is known in the literature as the *Harris-Todaro condition*.

The level of the urban population that satisfies the Harris-Todaro condition,  $n_u^*$ , is attained from the solution of the equation resulting from substitution of equations (3), (5), (6), and (7) in (8):

$$(8-a) \quad \left( \frac{n_m^*}{n_u^*} \right) \bar{\omega}_m - g'(1 - n_u^*) \rho(x_m^*/g(1 - n_u^*)) = 0.$$

Figure 1 illustrates this solution. Note that the position of the curve labeled  $\omega_a = g'(1 - n_u) \rho(x_m^*/g(1 - n_u))$  also depends on the minimum wage, since the relative price,  $p$ , depends upon the urban production,  $x_m^*$ . Combined with the diagram that represents the manufacturing demand curve for labor, we can also see urban unemployment, given by the difference between  $n_u^*$  and  $n_m^*$ .

[FIGURE 1]

Given the properties assumed for the functions  $g(\cdot)$  and  $\rho(\cdot)$ , we can establish the following proposition concerning the Harris-Todaro equilibrium.

**PROPOSITION 1:** *Suppose that  $g'(\cdot)$  and  $\rho(\cdot)$  are continuously differentiable functions in the interval  $[n_m^*, 1) \in \mathfrak{R}_{++}$ . The existence and uniqueness of the Harris-Todaro equilibrium with urban unemployment are guaranteed if*

$$(9) \quad \lim_{n_u \rightarrow 1^-} [g'(1 - n_u) \rho(x_m^*/g(1 - n_u))] > n_m^* \bar{\omega}_m$$

and

$$(10) \quad g'(1-n_m^*)\rho(x_m^*/g(1-n_m^*)) < \bar{\omega}_m.$$

PROOF:

Let  $\Phi(n_u) \equiv (n_m^*/n_u)\bar{\omega}_m - g'(1-n_u)\rho(x_m^*/g(1-n_u))$ . The function  $\Phi(n_u)$  is continuous in the interval  $[n_m^*, 1)$  because it is the sum of continuous functions. By conditions (9) and (10), it follows that  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) < 0$  and  $\Phi(n_m^*) > 0$ . If  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) = c < 0$ , with  $c$  a real constant, then for every  $\varepsilon > 0$  there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $|\Phi(n_u) - c| < \varepsilon$ . Specifically, let  $\varepsilon = -c > 0$ . Thus there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $|\Phi(n_u) - c| < -c$  and this last inequality implies  $\Phi(n_u) < 0$ . If  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) = -\infty$ , then for every  $A < 0$  there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $\Phi(n_u) < A$ . In both cases, we can take  $n_u = 1 - \frac{\delta}{2}$  such that  $\Phi\left(1 - \frac{\delta}{2}\right) < 0$ . Since  $\Phi(n_m^*) > 0$ ,  $\Phi\left(1 - \frac{\delta}{2}\right) < 0$  and  $\Phi$  is continuous in close interval  $\left[n_m^*, 1 - \frac{\delta}{2}\right] \subset [n_m^*, 1)$ , we can apply the intermediate value theorem and conclude that there is some  $n_u^* \in \left[n_m^*, 1 - \frac{\delta}{2}\right] \subset [n_m^*, 1)$  such that  $\Phi(n_u^*) = 0$ .

Moreover, considering (1), (2) and (5) we have

$$\frac{\partial \Phi}{\partial n_u} = -\frac{n_m^* \bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 < 0 \quad \text{for all } n_u \in [n_m^*, 1).$$

Hence, since  $\frac{\partial \Phi}{\partial n_u}$  is continuous in interval  $[n_m^*, 1)$ , there is only one  $n_u^* \in [n_m^*, 1)$  such that

$$\Phi(n_u^*) = 0.$$

By condition (9), as the numbers of rural workers tend to zero, both the labor productivity in this sector and the terms of trade tend to extreme high values. Thus the rural real wage becomes greater than the expected urban real wage. On its part, condition (10) establishes that, at the value for which the minimum wage restriction is binding, the real rural wage must be smaller than the expected urban real wage, which in the present case equals the exogenously determined minimum wage.

Harris and Todaro (1970, p. 129), in order to evaluate the long run equilibrium, postulate a mechanism of adjustment that is based on the following function of sign preservation:

$$(11) \quad \dot{n}_u = \psi(\omega_u^e - \omega_a), \quad \text{with } \psi'(\cdot) > 0 \text{ and } \psi(0) = 0.$$

Thus, while  $\omega_u^e > \omega_a$ ,  $\dot{n}_u > 0$  will be the result, and only if  $\omega_u^e = \omega_a$ ,  $\dot{n}_u = 0$  will occur. The differential equation that governs the state transition in the model of Harris and Todaro is obtained by substituting the left-hand side of (8-a) in (11):

$$(12) \quad \dot{n}_u = \psi\left((n_m^*/n_u)\bar{\omega}_m - g'(1-n_u)\rho(x_m^*/g(1-n_u))\right).$$

It should be stressed that the state space for (11) is the real interval  $[n_m^*, 1)$ , which is positively invariant.<sup>7</sup>

Based on this *postulated* adjustment process, Harris and Todaro (1970, p. 138-139) show that the long run equilibrium is globally stable.<sup>8</sup> This means that the economy of Harris and Todaro would tend to long run equilibrium with unemployment in the urban sector generated by the presence of a relatively high minimum wage.

## II. Migration of Workers as a Replicator Dynamics in a Milieu of Bounded Rationality

We begin our analysis by going back to the situation used by Harris and Todaro in their analysis of the rural-urban migratory process. We assume that, between consecutive periods of production, each worker decides to which sector he will supply his labor force. We exclude, by assumption, the possibility that a worker may simultaneously supply his labor force to both sectors. Hence, only two strategies are admitted for the worker: stay in the sector in which he was (employed or not) during the previous production period or migrate to the other sector.

As in the Harris and Todaro model, we will assume that the population of workers is kept constant and equal to  $\bar{n}$ , normalized to unity. In a given moment, there are, in population

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<sup>7</sup> Considering (10) in  $n_u = n_m^*$ , this results in  $\dot{n}_u > 0$ . Given condition (9) and the fact that  $\lim_{n_u \rightarrow 1^-} (n_m^*/n_u)\bar{\omega}_m = n_m^*\bar{\omega}_m$ , it follows that  $\lim_{n_u \rightarrow 1^-} \dot{n}_u < 0$ .

<sup>8</sup> This can be seen from (12). Considering that  $\psi'(\cdot) > 0$  and  $g''(\cdot) < 0$ , one concludes that:

$$\frac{\partial \dot{n}_u}{\partial n_u} = \psi'(\cdot) \left\{ -\frac{n_m^*\bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 \right\} < 0, \quad \text{for } n_m^* \leq n_u < 1.$$

units,  $n_u$  workers in the urban sector and  $n_r = 1 - n_u$ , in the rural sector. A fraction  $v_u$  of workers from the urban sector is employed and  $1 - v_u$ , unemployed. Thus there are  $n_u v_u$  employed workers in the urban sector, with  $n_u(1 - v_u)$  unemployed. Contrarily to Harris and Todaro, we assume the possibility of rural unemployment. Consequently, we may beforehand have a fraction  $v_r$  of rural workers employed, with the remaining rural workers unemployed. In units of population,  $n_r v_r$  workers from the rural sector are employed and  $n_r(1 - v_r)$ , unemployed.

Given that  $n_u + n_r = 1$ , it is enough to consider the evolution of one of these state variables. We choose the number of workers of the urban sector. The change in the urban sector population, for an infinitesimal time interval, is given by the difference between the estimated quantity of immigrant workers to the urban sector and the estimated quantity of emigrant workers from this sector to the rural sector:

$$(13) \quad \dot{n}_u = \text{urban sector inflow} - \text{urban sector outflow}$$

If a worker from a given sector compares his income with another worker from the same sector, he does not get information on the other market and thus he has no basis for evaluating his choice of strategy. In such a situation, we assume that the worker will not change his strategy. Comparison of income among workers from different markets will be taken as a necessary condition, even though not sufficient, for a migration to ensue.

In Table 1, we list the possibilities of comparisons of possible incomes in the economy. Event 1 stands for the following case: an employed worker in the rural sector and an employed worker in the urban sector measure up their incomes. Since in a given moment there are  $(1 - n_u)v_r$  workers employed in the rural sector and  $n_u v_u$  workers employed in the urban sector, measured in population units, then the estimated number of employed rural workers that will compare their incomes with the incomes of employed urban workers is  $(1 - n_u)v_r n_u v_u$ . The other events shown in Table 1 may be analogously interpreted.

[TABLE 1]

#### A. Rural-Urban Migratory Flows

The comparison on the part of a rural worker of his income with the income of an urban sector worker is only a necessary condition for him to change his strategy. Another necessary condition for change of strategy is that the real income in the urban sector ( $y_u$ ) be greater than the real income in the rural sector ( $y_r$ ). In a word, a rural worker is a *potential immigrant* to the urban sector whenever the comparison of incomes yields  $y_u > y_r$ .

We suppose that, satisfied the necessary conditions put above, *effecting* a strategy change will depend upon the difference between incomes of workers in rural e urban sectors. The higher is this difference in income the higher is the incentive for the rural worker to look for a new job in the urban sector. Put differently, if  $y_u - y_r > 0$ , the proportion of workers of the rural sector that will actually migrate is assumed to be directly proportional to the difference in income  $y_u - y_r$ .<sup>9</sup>

In Table 1, we summarize the income gaps that may appear in each possible comparison. The income of workers of the  $i^{th}$  sector was separated in two components: one is the real wage  $\omega_i$  (null when the worker is unemployed) and the other, the sum of incomes unconnected to the direct sale of his labor force,  $\bar{\epsilon}_i$ , assumed exogenous and hereafter named *non-wage income*.<sup>10</sup>

From Table 1, we can estimate the migratory flow of workers to the urban sector. When employed rural workers relate their income to the one of the employed urban workers (event 1) and  $y_u - y_r = (\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r) \leq 0$ , then the quantity of immigrant rural workers will be zero. However, when this comparison yields  $y_u - y_r = (\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r) > 0$ , the estimated inflow of employed workers in the urban sector is directly proportional to this differential in income (normalized). In a compact way, the subsequent expression gives both possibilities:

$$(14) \quad (1 - n_u)v_r n_u v_u \max\{(\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r), 0\}.$$

When employed rural workers compare their income with unemployed urban workers (event 2), there will be immigration to the urban sector provided the non-wage income at the urban sector is high enough as weighed against the expected total income in the rural sector:

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<sup>9</sup> In the theory of evolutionary games, this assumption may have several microeconomic foundations. See the appendix to this paper.

<sup>10</sup> The non-wage income may be generalized to include net benefits of the public budget plus other usual sources of income that are studied in the literature on migration. Thus, even under unemployment in both areas, it may be worth the effort to migrate to the urban sector.

$$(15) \quad (1 - n_u)v_r n_u (1 - v_u) \max\{\bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\}.$$

The same type of comparison is valid for events 3 and 4. Provided  $\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r > 0$ , the proportion of rural workers that effectively immigrate is directly proportional to this income difference:

$$(16) \quad (1 - n_u)(1 - v_r)n_u v_u \max\{\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}.$$

Likewise, an encounter of unemployed rural workers with unemployed urban workers, combined with appropriate differential in incomes will result in an inflow of workers to the urban sector given by

$$(17) \quad (1 - n_u)(1 - v_r)n_u (1 - v_u) \max\{\bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}.$$

By aggregating expressions (14), (15), (16) and (17), we get the expected migratory flow from the rural to the urban sector:

$$(18) \quad (1 - n_u)n_u [v_r v_u \max\{\omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\} + v_r (1 - v_u) \max\{\bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\} \\ + (1 - v_r)v_u \max\{\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\} + (1 - v_r)(1 - v_u) \max\{\bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}].$$

### B. Urban-Rural Migratory Flows

A worker from the urban sector becomes a *potential emigrant* only if he compares his income with the income of a worker from the rural sector and discovers that  $y_r = \omega_r + \bar{\varepsilon}_r > \omega_u + \bar{\varepsilon}_u = y_u$ . As before, four events are possible in this pairwise comparison of incomes, according to the situation of employment of each worker in the urban and the rural sectors.

Take, for example, event 1. As employed urban workers contrast their income with the income of their equals as to the employment situation in the rural area, two results are possible. If  $y_u - y_r = \omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r) \geq 0$ , no employed urban worker leaves his area. However, if  $y_u - y_r = \omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r) < 0$ , the migratory outflow from the urban area will be directly proportional to this difference in income, or:

$$(19) \quad -n_u v_u (1-n_u) v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\}.$$

The other three events are interpretable likewise. Thus, from event 3, the result of the meeting of an employed urban worker with unemployed rural workers may be described by:

$$(20) \quad -n_u v_u (1-n_u)(1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\}$$

Event 2, when unemployed urban workers come across employed workers from the rural sector, results in:

$$(21) \quad -n_u (1-v_u)(1-n_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\}.$$

At last, in event 4, unemployed workers from both areas meet and the corresponding expected migration is given by:

$$(22) \quad -n_u (1-v_u)(1-n_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}$$

By aggregating expressions (19) to (22), we attain the expected migratory outflow from the urban to the rural sector:

$$(23) \quad -n_u (1-n_u) [v_u v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_u (1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + (1-v_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + (1-v_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}].$$

### C. Replicator Dynamics

Given these possibilities of migration for the workers, the change in the proportion of workers located in the urban sector is approximated by the difference between the rural-urban flow and the urban-rural flow, i.e., by the difference between (18) and (23):

$$(24) \quad \dot{n}_u = n_u (1-n_u) [v_r v_u \max\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_r (1-v_u) \max\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} \\ + (1-v_r) v_u \max\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} + (1-v_r)(1-v_u) \max\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + v_u v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_u (1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + (1-v_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + (1-v_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}].$$

Given that  $\max\{d, 0\} + \min\{d, 0\} = d$  for any real constant  $d$ , the equation (24) may be simplified to:

$$(24-a) \quad \dot{n}_u = n_u(1 - n_u)[(v_u \omega_u + \bar{\epsilon}_u) - (v_r \omega_r + \bar{\epsilon}_r)].$$

This replicator dynamics formally mirrors the intuition that the urban labor force proportion increases, stays put, or lowers while the average income in this market  $(v_u \omega_u + \bar{\epsilon}_u)$  is, respectively, higher, equal, or lower than in the rural sector  $(v_r \omega_r + \bar{\epsilon}_r)$ .

Optionally, by simple algebraic handling, we get the replicator dynamics (24-b):

$$(24-b) \quad \dot{n}_u = n_u \{ (v_u \omega_u + \bar{\epsilon}_u) - [n_u (v_u \omega_u + \bar{\epsilon}_u) + (1 - n_u)(v_r \omega_r + \bar{\epsilon}_r)] \}.$$

The expression within brackets is the average real income of the economic system as a whole. Consequently, the supply of labor to the urban sector tends to expand if and only if the expected income in this market is superior to the average real income of the economy.

### III. The Harris-Todaro Condition as a Resultant Property of an Evolutionary Game

We have seen in section 2 that the Harris-Todaro model assumes that prices in the rural sector are perfectly flexible, which implies full employment there. This means that  $v_r(t) = 1$  for all  $t \geq t_0$ , with  $t_0$  an arbitrary initial point in time. Moreover, in the urban sector, for a minimum wage, the employment level is  $n_m^* \leq n_u$ . As a result, the probability that a worker finds a job in the urban sector is inversely related to the size of the population in this sector, i.e.,  $v_u = n_m^* / n_u$ . Given that in the Harris and Todaro model non-wage income ( $\bar{\epsilon}_u = \bar{\epsilon}_r = 0$ ) are not taken into account, we have that  $y_u = \omega_u = \bar{\omega}_m$  and  $y_r = \omega_a$ . Given these assumptions, the replicator dynamics, as expressed by (24-a), becomes:

$$(25) \quad \dot{n}_u = n_u(1 - n_u) \left( \frac{n_m^*}{n_u} \bar{\omega}_m - \omega_a \right).$$

The Harris-Todaro condition, as given by (8), is an equilibrium condition in the replicator dynamics of (25). Therefore, the size of the urban population at the long run equilibrium of Harris and Todaro,  $n_u^*$ , is also an equilibrium value from equation of (25). This equilibrium, from the viewpoint of the evolutionary game theory, is an *equilibrium of mixed strategy*, since rural workers keep moving to the urban sector and vice-versa, although the respective sizes of the urban and rural populations are stable. What happens in this kind of

equilibrium is that there is equality between the flow of rural workers that migrate to the urban sector and the flow of urban workers that migrate to the rural sector. In other words, in this mixed strategy equilibrium there is a macroequilibrium generated by intersector migration flows that counterbalance each other.

Worth pointing out is the fact that the replicator dynamics (25), by itself replaces the postulated mechanism of adjustment of Harris and Todaro, which is based on a sign preserving function, i.e., the replicator dynamics substitutes the differential equation (11). Said differently, the mechanism of labor allocation of Harris and Todaro comes out from the interaction of heterogeneous agents in a milieu of bounded rationality.

Now we have just to study the replicator dynamics of (25). Substituting the left-hand side of (8-a) into (25), we get:

$$(25-a) \quad \dot{n}_u = n_u(1 - n_u)\Phi(n_u),$$

remembering that  $\Phi(n_u) \equiv (n_m^*/n_u)\bar{\omega}_m - g'(1 - n_u)\rho(x_m^*/g(1 - n_u))$ .

**PROPOSITION 2:** *The evolutionary dynamics whose state transition is governed by (25-a) converge to the Harris-Todaro (mixed strategy) equilibrium with urban unemployment for any initial condition such that  $n_u(t_0) \in [n_m^*, 1) \in \mathfrak{R}_{++}$ .*

**PROOF:**

Given that  $n_u(1 - n_u) > 0$  for all  $n_u \in [n_m^*, 1)$ , the behavior of the state variable  $n_u$  depends on  $\Phi(n_u)$ , i.e., on the expected wage differential. Since  $g(\cdot) > 0$ ,  $g''(\cdot) < 0$ ,  $\rho(\cdot) > 0$ , and  $\rho'(\cdot) > 0$  for all  $n_u \in [n_m^*, 1)$ , the impact of a variation in urban population on the expected wage differential is given by:

$$(26) \quad \frac{\partial \Phi}{\partial n_u} = -\frac{n_m^* \bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 < 0, \text{ for all } n_u \in [n_m^*, 1).$$

Thus in the interval  $[n_m^*, 1)$ , if  $n_u \in [n_m^*, n_u^*)$ , then  $\Phi(n_u) > 0$  and if  $n_u \in (n_u^*, 1)$ , then  $\Phi(n_u) < 0$ . Hence we deduce that for any initial condition  $n_u(t_0) \in [n_m^*, 1)$  the economic system converges to the Harris-Todaro (mixed strategy) equilibrium with urban unemployment,  $n_u^*$ .

If the urban population is below this equilibrium, i.e.,  $n_u \in [n_m^*, n_u^*]$ , then the urban employment rate is too high, the marginal productivity of rural labor is too low and the terms of trade are biased towards manufactured goods. This situation gives rise to an average real wage in the urban sector greater than the real wage in the rural sector. Such a favorable income differential for the urban sector generates a positive net rural-urban migratory flow. This keeps going until its negative effect on the rate of urban employment and its positive effect on the marginal productivity of rural labor and on the terms of trade equalize average real wage between the two sectors. On the contrary, when the urban population is above the level of the Harris-Todaro mixed strategy equilibrium,  $n_u \in (n_u^*, 1)$ , the urban employment rate is too low, the marginal productivity is too high and the terms of trade are biased towards agricultural goods. Now the average real wage in the urban sector is lower than the real wage in the rural sector. This leads to a reversal in rural-urban migration, constraining the migratory movement and taking the system back to the mixed strategy equilibrium. To put in a nutshell, the Harris-Todaro condition emerges as the outcome of the interaction among heterogeneous workers that are worried à la Keynes with their relative wages in conditions of bounded rationality.

#### **IV. Conclusions**

In this paper, we developed an evolutionary game model, more specifically a replicator dynamics, which formalizes the sector allocation of labor as a process of imitation/learning in a milieu of bounded rationality. The Harris-Todaro condition comes out as a spontaneous upshot of interaction among heterogeneous workers.

In the model we propose here, due to the separation of income in two parts – wage and non-wage income – there is room for studying the role of public policies that influence the non-wage component of income and therefore influence the migratory dynamics of Harris and Todaro. Analyzing this dynamics is directly relevant to a better understanding of the process of urban concentration.

APPENDIX:

AN ALTERNATIVE DERIVATION OF THE REPLICATOR DYNAMICS (24-A):

The evolutionary game models based on an *imitation process* are formed by two essential pieces as highlighted by Weibull (1995, p. 152): "the *time rate* at which agents review their strategy choice" and "the *choice probabilities* of a reviewing agent." Let  $x_i$  be the proportion of individuals from the population that opt for the strategy  $s_i$  (*type i agents*), among  $h$  possible strategies and  $x = (x_1, x_2, \dots, x_i, \dots, x_h)$  be the distribution of strategies in the population. The (average) rate of revision of a type  $i$  agent refers to the average number of times this agent revises his strategy per time interval, and may be taken as function of the state of the population,  $r_i(x)$ . This rate of revision may be considered as an *arrival rate* of a Poisson process, i.e., a realization of Poisson process. Let us assume that the frequency with which an agent reconsiders his strategy in a given time interval does not affect the frequency of revisions of the other agents, which is the same as saying that the Poisson process of individual revisions are statistically independent. We then conclude that the aggregate rate of revision of the subpopulation given by type  $i$  agents is by itself a Poisson process with an average rate of revision  $x_i r_i(x)$ . If changes in individual strategies are statistically independent random variables, then the flow of agents from the type  $i$  subpopulation to the type  $j$  subpopulation is an aggregate Poisson process with the arrival rate  $x_i r_i(x) p_i^j(x)$ , where  $p_i^j(x)$  is the probability of a type  $i$  agent becoming a type  $j$ . The choice probabilities of a type  $i$  agent form a probability distribution, represented by  $p_i(x) = (p_i^1(x), \dots, p_i^j(x), \dots, p_i^h(x))$ , naturally with  $\sum_{j=1}^h p_i^j(x) = 1$ .

In the case of only two possible strategies,  $h = 2$ , we have  $x_1 + x_2 = 1$ . Thus we need to consider only one state variable. Without loss of generality, we will take  $x_1$  as the reference state variable, so the vector of distribution of strategies will be perfectly determined once  $x_1$  is given, because  $x = (x_1, x_2) = (x_1, 1 - x_1)$ . The inflow to the subpopulation of type  $i$  agents can be approximated by

$$(27) \quad x_2 r_2(x) p_2^1(x)$$

and the outflow by

$$(28) \quad x_1 r_1(x) p_1^2(x).$$

The rate of variation of subpopulation 1 will be determined by the difference between inflow and outflow:

$$(29) \quad \dot{x}_1 = x_2 r_2(x) p_2^1(x) - x_1 r_1(x) p_1^2(x).$$

As explained by Weibull (1995, sect. 4), there are several assumptions pertaining to the revision rate and to the probability of choice that may be adopted. In section 3 above, we implicitly assumed that all reviewing agents reassess their choice together and just once per time interval, or

$$(30) \quad r_1(x) = r_2(x) = 1$$

As in section 3, we assume a *direct imitation process* (Weibull, 1995, p. 155-158), i.e., each reviewing agent randomly meets another agent so that the probability of a reviewing agent encountering an agent that follows strategy  $s_1$  is  $x_1$  and one that follows  $s_2$  is  $x_2 = 1 - x_1$ . As done by Weibull (1995, p. 155-158), we suppose that, after the reviewing agent meets another agent, he detects, with some noise, his own payoff,<sup>11</sup>  $\pi_i(x) + \alpha_i$ , and the payoff linked to the strategy of the other agent,  $\pi_j(x) + \alpha_j$ . Since  $\alpha_i$  and  $\alpha_j$  are stochastic variables, the difference  $\alpha_i - \alpha_j$  is also a stochastic variable, whose cumulative distribution function  $F : R \rightarrow [0,1]$  is assumed to be continuously differentiable.

The type  $i$  reviewing agent becomes a type  $j$  agent if and only if the *observed* result of the latter overcomes the *observed* result of the former, i.e.,  $\pi_j(x) + \alpha_j > \pi_i(x) + \alpha_i$ , or equivalently,  $\alpha_i - \alpha_j < \pi_j(x) - \pi_i(x)$ . The probability that this happens is given by the image of the cumulative distribution function at the point  $\pi_j(x) - \pi_i(x)$ , that is,  $F(\pi_j(x) - \pi_i(x))$ . This function may be interpreted in other manners. Weibull (1995, p. 157), for example, gives an alternative rendition to the variables  $\alpha_i$  and  $\alpha_j$ . They may be seen as idiosyncratic differences between preferences of the agents. As a result,  $\pi_i(x) + \alpha_i$  and  $\pi_j(x) + \alpha_j$  become the true payoffs from strategies  $s_i$  and  $s_j$ , and the choice probabilities become the result from the differences between the strategies of agents, instead of individual observational errors by agents with similar preferences. Nachbar (1990) gives an alternative

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<sup>11</sup>  $\pi_i(x)$  is the expected result for an agent that opts for a strategy  $s_i$  when the population state is  $x = (x_1, x_2) = (x_1, 1 - x_1)$ .

interpretation to function  $F(\cdot)$ , in which it reflects a change cost,  $c$ , not given in a deterministic form and instantly paid. In fact,  $F(\cdot)$  is assumed as a cumulative distribution function that furnishes the probability that  $c \leq \pi_j(x) - \pi_i(x)$ . Hence, when a type  $i$  reviewing agent compares his payoff with a type  $j$ , he only changes his strategy if the cost of changing is lower or equal to the difference between the payoffs. In short, these authors include observational errors, distinctions in preferences, or stochastic adjustment costs in order to give a microeconomic foundation to the imitation-based selection dynamics.

Summarizing, the probability that an agent of type  $i$  meets an agent of type  $j$  is  $x_j$ , and the probability that a type  $i$  agent, given that he has met a type  $j$ , choose a strategy  $s_j$ , is  $F(\pi_j(x) - \pi_i(x))$ . Assuming that these two events are statistically independent, then the probability distribution function for the choice of a type  $i$  agent may be so expressed:

$$(31) \quad p_i(x) = (p_i^j(x), p_i^i(x)) = (x_j F(\pi_j(x) - \pi_i(x)), 1 - x_j F(\pi_j(x) - \pi_i(x))),$$

for  $j \neq i$  and  $i, j = 1, 2$ . Substituting (30) and (31) into (29), we get:

$$(32) \quad \dot{x}_1 = x_1(1 - x_1)[F(\pi_1(x) - \pi_2(x)) - F(\pi_2(x) - \pi_1(x))].$$

Now suppose that the stochastic variable  $\alpha_i - \alpha_j$  be uniformly distributed, i.e., that its probability density function be:

$$(33) \quad f(\alpha_i - \alpha_j) = \begin{cases} \beta, & -a \leq \alpha_i - \alpha_j \leq a \\ 0, & \text{elsewhere} \end{cases},$$

with support  $[-a, a]$ , in which  $a = \max\{|\pi_1(x) - \pi_2(x)|, \forall x \in \Delta^2\}$ , with  $\beta > 0$  being a real constant. We then get the following cumulative distribution function:

$$(34) \quad F(\pi_j(x) - \pi_i(x)) = \frac{[\pi_j(x) - \pi_i(x)] + a}{2a}, \text{ with } j \neq i.$$

Inserting (34) into (32), we attain:

$$(35) \quad \dot{x}_1 = (1/a)x_1(1 - x_1)[\pi_1(x) - \pi_2(x)].$$

In the replicator dynamics (24-a), from section 3, we have that  $x_1 = n_u$  and  $x_2 = n_r = 1 - n_u$ , thus the type 1 agent is the urban worker and the type 2 agent is the rural worker. The expected payoff of a type 1 agent (urban worker) becomes:

$$(36) \quad \pi_1(x) = v_u \omega_u + \bar{\epsilon}_u,$$

and the respective expected payoff of a rural worker turns into:

$$(37) \quad \pi_2(x) = v_r \omega_r + \bar{\epsilon}_r.$$

For this reason, by considering (36) and (37), the replicator dynamics may be written as:

$$(38) \quad \dot{n}_u = (1/a)n_u(1-n_u)[(v_u \omega_u + \bar{\epsilon}_u) - (v_r \omega_r + \bar{\epsilon}_r)],$$

in which

$$a = \max\{|\pi_1(x) - \pi_2(x)|, \forall x \in \Delta^2\} = \max\{|v_u \omega_u + \bar{\epsilon}_u - (v_r \omega_r + \bar{\epsilon}_r)|, \forall n_u \in (0,1) \in \mathfrak{R}\}.$$

Equation (38) is the replicator dynamics (24-a), except for the scalar  $1/a$ , which does not alter the qualitative structure of the solutions of the replicator dynamics under inquiry. As a result, (24-a) may be interpreted as a normalized version of (38). Normalizing  $a = 1$ , we can say that the income difference in (24-a) may now be expressed in units of the maximum differential in average income that is a priori possible.

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Table 1 – Income differential and number of estimated comparisons between workers of urban and rural sectors, according to employment situation

		Event			
		1	2	3	4
Urban sector worker	<b>Situation</b>	Employed	Unemployed	Employed	Unemployed
	<b>Income</b> ( $y_u$ )	$\omega_u + \bar{\epsilon}_u$	$\bar{\epsilon}_u$	$\omega_u + \bar{\epsilon}_u$	$\bar{\epsilon}_u$
	<b>Sub-population</b>	$n_u v_u$	$n_u (1 - v_u)$	$n_u v_u$	$n_u (1 - v_u)$
Rural sector worker	<b>Situation</b>	Employed	Employed	Unemployed	Unemployed
	<b>Income</b> ( $y_r$ )	$\omega_r + \bar{\epsilon}_r$	$\omega_r + \bar{\epsilon}_r$	$\bar{\epsilon}_r$	$\bar{\epsilon}_r$
	<b>Sub-population</b>	$(1 - n_u)v_r$	$(1 - n_u)v_r$	$(1 - n_u)(1 - v_r)$	$(1 - n_u)(1 - v_r)$
<b>Income diferencial</b> ( $y_u - y_r$ )		$(\omega_u + \bar{\epsilon}_u) - (\omega_r + \bar{\epsilon}_r)$	$\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r)$	$\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r$	$\bar{\epsilon}_u - \bar{\epsilon}_r$
<b>Number of estimated comparisons</b>		$n_u v_u (1 - n_u)v_r$	$n_u (1 - v_u)(1 - n_u)v_r$	$n_u v_u (1 - n_u)(1 - v_r)$	$n_u (1 - v_u)(1 - n_u)(1 - v_r)$

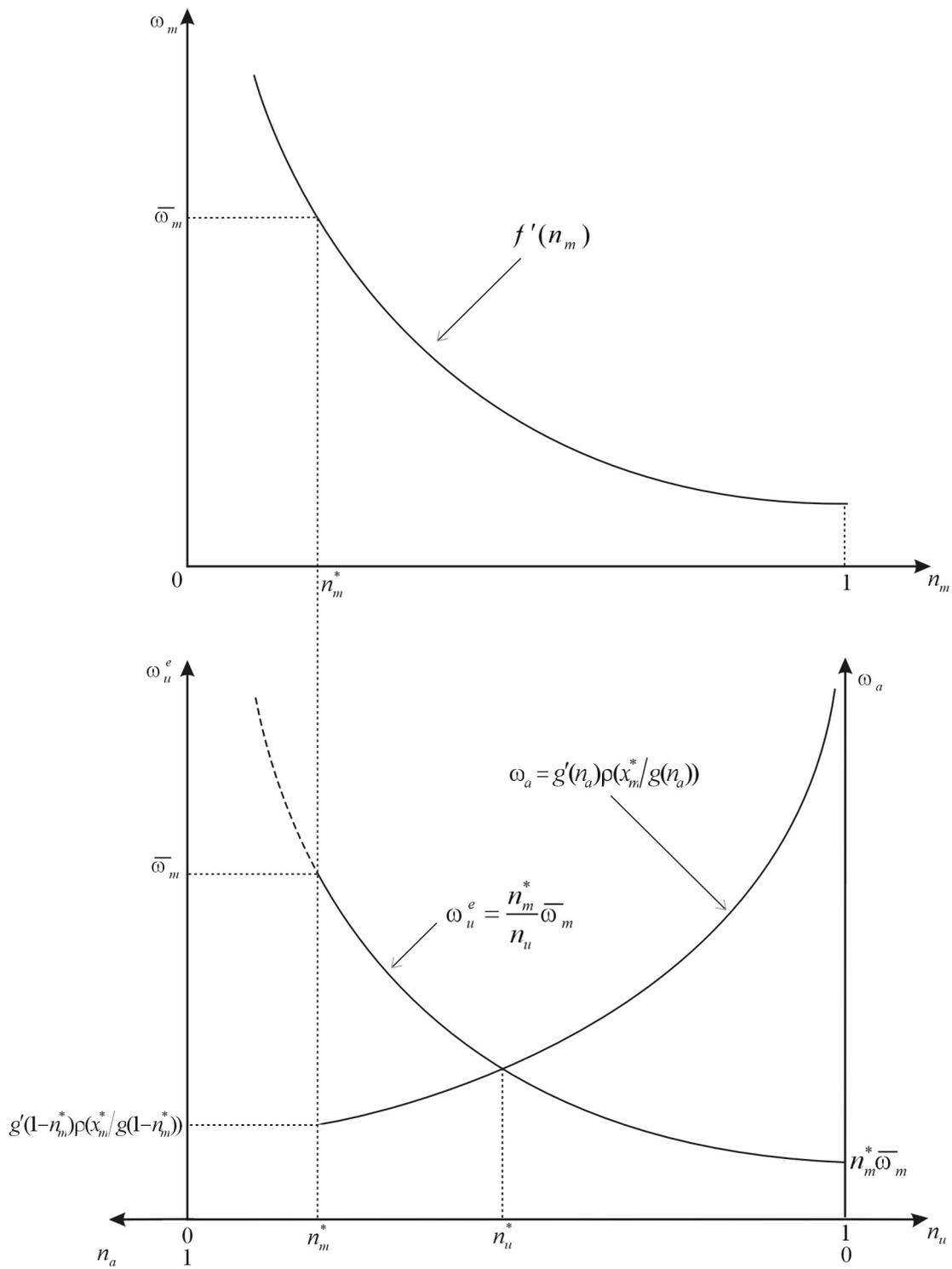


Figure 1 – Harris-Todaro model with endogenous terms of trade

# The Harris-Todaro Labor Allocation Mechanism as an Evolutionary Game

By Jaylson Jair da Silveira and João Rogério Sanson\*

## Abstract

The Harris-Todaro model is reinterpreted under an evolutionary game approach, including the behavior of migrants under bounded rationality in a process of imitation or learning. A first feature is an improvement upon Harris-Todaro by replacing the sign-preserving function of the adjustment mechanism by a replicator dynamics, which is deduced from the assumptions about the interaction process among heterogeneous workers in a bounded rationality context. A second feature is an interpretation of the equilibrium condition as a mixed strategy equilibrium of an evolutionary game. The Harris-Todaro equilibrium condition then appears as a spontaneous result of the model.

**Key words:** migration, labor market, evolutionary game, bounded rationality

**JEL codes:** J6, O15, C7

Developing countries generally show some kind of dualism in their labor markets, be it with respect to production structure (traditional and modern), to geographical location (rural and urban), to legal nature of the activities (formal and underground), or to the composition of the labor force (skilled and non-skilled). As shown by Agénor and Montiel (1996, p. 63), duality in the labor market implies a segmentation, i.e., a situation in which identical workers earn different wages depending on where they are employed.

Todaro (1969) has built up a seminal model to analyze rural-urban migration in a developing country, extending and formalizing ideas from various authors that followed Lewis (1954). As the main assumption of the model, the decision of the rural worker to migrate depends upon expected differential wage. In Michael Todaro's analysis, the decision to move is seen as an investment decision tied to expected net returns. These expected returns crucially depend upon the probability of getting a job in the modern, or formal, sector versus a job at the traditional urban sector, also called underground or informal. The model takes this probability as endogenous and influenced by the creation of urban employment and the

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number of unemployed urban workers. The objective of the model is to show how the employment rate tends to an equilibrium below full employment even in the long run. Harris and Todaro (1970) is a general equilibrium analysis in which the artificial upholding of the wage differential between the rural and urban sectors leads to an inefficient equilibrium. Although the informal sector is now excluded from the model, the concept of expected wage is kept. What determines the expected wage is employment or unemployment in the urban sector.

Despite defending the principle that workers take into consideration the present value of the expected real income flow, Todaro (1969) as well as Harris and Todaro (1970) in fact work with the postulate of myopic expectations. Todaro (1969, sect. IV) formally obtains his main conclusions from the assumption that the time horizon of the workers is only one period. Thus, in their model, the differential in real income for time  $t$  appears in place of the differential of the present value of the flow of expected real income along two or more periods. This is done in the adjustment mechanism associated with the aggregate supply of workers to the urban sector. Todaro (1969, p. 143, n. 10) justifies this by saying that this "... assumption is made necessary by mathematical convenience but is *in fact probably a more realistic formulation in terms of actual decision making in less developed nations ...*" [Our italics.] We consider this implicit idea of bounded rationality of the workers, and propose a model based on the tools of the theory of evolutionary games.<sup>12</sup>

In our model, the migratory movement of workers is interpreted as a process of imitation/learning in an environment of bounded rationality.<sup>13</sup> With the help of an evolutionary game, we deduce a replicator dynamics that replaces the adjustment mechanism *postulated* both in Todaro (1969) and Harris and Todaro (1970). Starting with their same general equilibrium structure, but including it in our model of evolutionary game, we show that their classical results appear as a consequence of the interaction among economic agents that try to get higher wages in a context of bounded rationality.

## **I. The Harris-Todaro Analysis of the Rural-Urban Migratory Movement**

Harris and Todaro (1970) study the migration of workers in a two-sector economic system, called *rural* and *urban*. These sectors differ by the kind of goods they produce and by

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<sup>12</sup> On this theory, see Weibull (1995), Vega-Redondo (1996), Samuelson (1997), and Hofbauer and Sigmund (1998).

the technology of production. The rural sector specializes in the production of an agricultural good whose productive process is described by the following production function:<sup>14</sup>

$$(1) \quad x_a = g(n_a), \quad \text{with } g'(n_a) > 0 \text{ and } g''(n_a) < 0,$$

in which  $x_a$  is the production level of the agricultural good and  $n_a$  is the input quantity of workers (in units of population) to the rural sector. The endowment of land and the stock of capital of this sector are given for the period of analysis.

Similarly, the urban sector has the following production function:

$$(2) \quad x_m = f(n_m), \quad \text{with } f'(n_m) > 0 \text{ and } f''(n_m) < 0,$$

where  $x_m$  is the production level of the manufactured good and  $n_m$  is the quantity of workers (in units of population) used in the production of manufactured goods. The stock of capital of this sector is also given during the period of analysis.

Both goods and labor markets are perfectly competitive. However, there is segmentation in the labor market. For the rural sector, the real wage, perfectly flexible, is equal to the marginal productivity of labor:

$$(3) \quad \omega_a = g'(n_a)p,$$

where  $\omega_a$  is the real wage and  $p$  is the price of the agricultural good, both expressed in terms of the manufactured good.<sup>15</sup>

For the urban sector, besides perfectly competitive markets, it is assumed a minimum wage,  $\bar{\omega}_m$ , fixed at a level above equilibrium in this labor market. Formally:

$$(4) \quad \bar{\omega}_m = f'(n_m^*) \text{ such that } n_m^* \leq n_u,$$

where  $n_u$  is the quantity of workers (in population units) in the urban sector.

As to the terms of trade between the two sectors, it is assumed that the relative price varies according to the relative scarcity between them, i.e.,

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<sup>13</sup> The model is based on one initially presented in Silveira (2001), where it was used to represent the competition among workers along business cycles.

<sup>14</sup> Our notation is slightly different from the original one of Harris and Todaro (1970). We work, without loss of generality, with the number of workers normalized in units of total population, since this is the adequate variable for the evolutionary game that we will develop in the next section.

<sup>15</sup> Thus the manufactured good is taken as the numeraire.

$$(5) \quad p = \rho(x_m/x_a), \text{ with } \rho'(x_m/x_a) > 0.$$

Within the period of analysis, the total population of workers in the economy is given at a level  $\bar{n} > 0$ . Without loss of generality, we may normalize the population size to  $\bar{n} = 1$ . Since by assumption there are only two sectors and rural prices are wholly flexible,<sup>16</sup> then at any time the following equality is verified:

$$(6) \quad n_a + n_u = 1.$$

With these assumptions, there is a general equilibrium structure that determines, for a given urban population of workers, a relative price structure,  $(\omega_a, p)$ , a sector allocation of labor,  $(n_a, n_m)$ , and the sector production,  $(x_a, x_m)$ . The urban sector minimum wage being fixed at  $\bar{\omega}_m$ , the firms of this sector, as price takers, maximize profits by employing  $n_m^*$  and producing  $x_m^*$ . In other words, given  $\bar{\omega}_m$ , by (4) we get  $n_m^*$ ; then we get  $x_m^*$  with the help of (2). Therefore, during the period of analysis, given the minimum wage, the employment level and production of the urban sector are unaltered.

Once the distribution of workers between the sectors of the economy is given, the real wage of the rural sector and the terms of trade between the sectors adjust until all the rural workers are employed and the full employment production level is reached. In other words, given  $n_u$ , we get  $n_a$  by (6). With  $n_a$  and (1), we obtain the full employment production of the rural sector,  $x_a$ . The sector production levels determine the terms of trade  $p$  through equation (5). Finally, the real wage of the rural sector in terms of the manufactured good,  $\omega_a$ , is obtained by (3), given  $n_a$  and  $p$ . In short, the vector  $(\omega_a, p, n_a, n_m^*, x_a, x_m^*)$  configures a temporary equilibrium or a shortrun, which is defined for a given vector of exogenous variables<sup>17</sup> and a given urban population of workers  $n_u$ . This temporary equilibrium will be disturbed when a redistribution of the population of workers between sectors happens, i.e., when there is a migration of workers that changes the urban population.

Harris and Todaro, for determining the long run equilibrium, argue that the rural workers reckon, in their decision on migrating to the urban area, the expected wage,  $\omega_u^e$ , defined as:

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<sup>16</sup> This means that there is full employment in the rural area.

<sup>17</sup> The endowment of land, the stocks of capital and the minimum wage.

$$(7) \quad \omega_u^e = \frac{n_m^*}{n_u} \bar{\omega}_m.$$

The key assumption of the model of Harris and Todaro is that there will be a migratory flow from the rural to the urban sector while the expected urban real wage is higher than in the rural sector. Thus, the *long run equilibrium*, i.e., the absence of a rural-urban migratory flow is established when the urban worker population reaches a level such that the expected urban real wage equates the rural real wage, i.e.:

$$(8) \quad \omega_u^e = \omega_a.$$

This equality is known in the literature as the *Harris-Todaro condition*.

The level of the urban population that satisfies the Harris-Todaro condition,  $n_u^*$ , is attained from the solution of the equation resulting from substitution of equations (3), (5), (6), and (7) in (8):

$$(8-a) \quad \left( \frac{n_m^*}{n_u^*} \right) \bar{\omega}_m - g'(1 - n_u^*) \rho(x_m^* / g(1 - n_u^*)) = 0.$$

Figure 1 illustrates this solution. Note that the position of the curve labeled  $\omega_a = g'(1 - n_u) \rho(x_m^* / g(1 - n_u))$  also depends on the minimum wage, since the relative price,  $p$ , depends upon the urban production,  $x_m^*$ . Combined with the diagram that represents the manufacturing demand curve for labor, we can also see urban unemployment, given by the difference between  $n_u^*$  and  $n_m^*$ .

[FIGURE 1]

Given the properties assumed for the functions  $g(\cdot)$  and  $\rho(\cdot)$ , we can establish the following proposition concerning the Harris-Todaro equilibrium.

**PROPOSITION 1:** *Suppose that  $g'(\cdot)$  and  $\rho(\cdot)$  are continuously differentiable functions in the interval  $[n_m^*, 1) \in \mathfrak{R}_{++}$ . The existence and uniqueness of the Harris-Todaro equilibrium with urban unemployment are guaranteed if*

$$(9) \quad \lim_{n_u \rightarrow 1^-} [g'(1 - n_u) \rho(x_m^* / g(1 - n_u))] > n_m^* \bar{\omega}_m$$

and

$$(10) \quad g'(1-n_m^*)\rho(x_m^*/g(1-n_m^*)) < \bar{\omega}_m.$$

PROOF:

Let  $\Phi(n_u) \equiv (n_m^*/n_u)\bar{\omega}_m - g'(1-n_u)\rho(x_m^*/g(1-n_u))$ . The function  $\Phi(n_u)$  is continuous in the interval  $[n_m^*, 1)$  because it is the sum of continuous functions. By conditions (9) and (10), it follows that  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) < 0$  and  $\Phi(n_m^*) > 0$ . If  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) = c < 0$ , with  $c$  a real constant, then for every  $\varepsilon > 0$  there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $|\Phi(n_u) - c| < \varepsilon$ . Specifically, let  $\varepsilon = -c > 0$ . Thus there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $|\Phi(n_u) - c| < -c$  and this last inequality implies  $\Phi(n_u) < 0$ . If  $\lim_{n_u \rightarrow 1^-} \Phi(n_u) = -\infty$ , then for every  $A < 0$  there is some  $\delta > 0$  such that, for all  $n_u$ , if  $1 - \delta < n_u < 1$  then  $\Phi(n_u) < A$ . In both cases, we can take  $n_u = 1 - \frac{\delta}{2}$  such that  $\Phi\left(1 - \frac{\delta}{2}\right) < 0$ . Since  $\Phi(n_m^*) > 0$ ,  $\Phi\left(1 - \frac{\delta}{2}\right) < 0$  and  $\Phi$  is continuous in close interval  $\left[n_m^*, 1 - \frac{\delta}{2}\right] \subset [n_m^*, 1)$ , we can apply the intermediate value theorem and conclude that there is some  $n_u^* \in \left[n_m^*, 1 - \frac{\delta}{2}\right] \subset [n_m^*, 1)$  such that  $\Phi(n_u^*) = 0$ .

Moreover, considering (1), (2) and (5) we have

$$\frac{\partial \Phi}{\partial n_u} = -\frac{n_m^* \bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 < 0 \quad \text{for all } n_u \in [n_m^*, 1).$$

Hence, since  $\frac{\partial \Phi}{\partial n_u}$  is continuous in interval  $[n_m^*, 1)$ , there is only one  $n_u^* \in [n_m^*, 1)$  such that

$$\Phi(n_u^*) = 0.$$

By condition (9), as the numbers of rural workers tend to zero, both the labor productivity in this sector and the terms of trade tend to extreme high values. Thus the rural real wage becomes greater than the expected urban real wage. On its part, condition (10) establishes that, at the value for which the minimum wage restriction is binding, the real rural wage must be smaller than the expected urban real wage, which in the present case equals the exogenously determined minimum wage.

Harris and Todaro (1970, p. 129), in order to evaluate the long run equilibrium, postulate a mechanism of adjustment that is based on the following function of sign preservation:

$$(11) \quad \dot{n}_u = \psi(\omega_u^e - \omega_a), \quad \text{with } \psi'(\cdot) > 0 \text{ and } \psi(0) = 0.$$

Thus, while  $\omega_u^e > \omega_a$ ,  $\dot{n}_u > 0$  will be the result, and only if  $\omega_u^e = \omega_a$ ,  $\dot{n}_u = 0$  will occur. The differential equation that governs the state transition in the model of Harris and Todaro is obtained by substituting the left-hand side of (8-a) in (11):

$$(12) \quad \dot{n}_u = \psi\left(\left(n_m^*/n_u\right)\bar{\omega}_m - g'(1-n_u)\rho(x_m^*/g(1-n_u))\right).$$

It should be stressed that the state space for (11) is the real interval  $[n_m^*, 1)$ , which is positively invariant.<sup>18</sup>

Based on this *postulated* adjustment process, Harris and Todaro (1970, p. 138-139) show that the long run equilibrium is globally stable.<sup>19</sup> This means that the economy of Harris and Todaro would tend to long run equilibrium with unemployment in the urban sector generated by the presence of a relatively high minimum wage.

## II. Migration of Workers as a Replicator Dynamics in a Milieu of Bounded Rationality

We begin our analysis by going back to the situation used by Harris and Todaro in their analysis of the rural-urban migratory process. We assume that, between consecutive periods of production, each worker decides to which sector he will supply his labor force. We exclude, by assumption, the possibility that a worker may simultaneously supply his labor force to both sectors. Hence, only two strategies are admitted for the worker: stay in the sector in which he was (employed or not) during the previous production period or migrate to the other sector.

As in the Harris and Todaro model, we will assume that the population of workers is kept constant and equal to  $\bar{n}$ , normalized to unity. In a given moment, there are, in population

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<sup>18</sup> Considering (10) in  $n_u = n_m^*$ , this results in  $\dot{n}_u > 0$ . Given condition (9) and the fact that  $\lim_{n_u \rightarrow 1^-} \left(n_m^*/n_u\right)\bar{\omega}_m = n_m^*\bar{\omega}_m$ , it follows that  $\lim_{n_u \rightarrow 1^-} \dot{n}_u < 0$ .

<sup>19</sup> This can be seen from (12). Considering that  $\psi'(\cdot) > 0$  and  $g''(\cdot) < 0$ , one concludes that:

$$\frac{\partial \dot{n}_u}{\partial n_u} = \psi'(\cdot) \left\{ -\frac{n_m^*\bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 \right\} < 0, \quad \text{for } n_m^* \leq n_u < 1.$$

units,  $n_u$  workers in the urban sector and  $n_r = 1 - n_u$ , in the rural sector. A fraction  $v_u$  of workers from the urban sector is employed and  $1 - v_u$ , unemployed. Thus there are  $n_u v_u$  employed workers in the urban sector, with  $n_u(1 - v_u)$  unemployed. Contrarily to Harris and Todaro, we assume the possibility of rural unemployment. Consequently, we may beforehand have a fraction  $v_r$  of rural workers employed, with the remaining rural workers unemployed. In units of population,  $n_r v_r$  workers from the rural sector are employed and  $n_r(1 - v_r)$ , unemployed.

Given that  $n_u + n_r = 1$ , it is enough to consider the evolution of one of these state variables. We choose the number of workers of the urban sector. The change in the urban sector population, for an infinitesimal time interval, is given by the difference between the estimated quantity of immigrant workers to the urban sector and the estimated quantity of emigrant workers from this sector to the rural sector:

$$(13) \quad \dot{n}_u = \text{urban sector inflow} - \text{urban sector outflow}$$

If a worker from a given sector compares his income with another worker from the same sector, he does not get information on the other market and thus he has no basis for evaluating his choice of strategy. In such a situation, we assume that the worker will not change his strategy. Comparison of income among workers from different markets will be taken as a necessary condition, even though not sufficient, for a migration to ensue.

In Table 1, we list the possibilities of comparisons of possible incomes in the economy. Event 1 stands for the following case: an employed worker in the rural sector and an employed worker in the urban sector measure up their incomes. Since in a given moment there are  $(1 - n_u)v_r$  workers employed in the rural sector and  $n_u v_u$  workers employed in the urban sector, measured in population units, then the estimated number of employed rural workers that will compare their incomes with the incomes of employed urban workers is  $(1 - n_u)v_r n_u v_u$ . The other events shown in Table 1 may be analogously interpreted.

[TABLE 1]

#### A. Rural-Urban Migratory Flows

The comparison on the part of a rural worker of his income with the income of an urban sector worker is only a necessary condition for him to change his strategy. Another necessary condition for change of strategy is that the real income in the urban sector ( $y_u$ ) be greater than the real income in the rural sector ( $y_r$ ). In a word, a rural worker is a *potential immigrant* to the urban sector whenever the comparison of incomes yields  $y_u > y_r$ .

We suppose that, satisfied the necessary conditions put above, *effecting* a strategy change will depend upon the difference between incomes of workers in rural e urban sectors. The higher is this difference in income the higher is the incentive for the rural worker to look for a new job in the urban sector. Put differently, if  $y_u - y_r > 0$ , the proportion of workers of the rural sector that will actually migrate is assumed to be directly proportional to the difference in income  $y_u - y_r$ .<sup>20</sup>

In Table 1, we summarize the income gaps that may appear in each possible comparison. The income of workers of the  $i^{th}$  sector was separated in two components: one is the real wage  $\omega_i$  (null when the worker is unemployed) and the other, the sum of incomes unconnected to the direct sale of his labor force,  $\bar{\epsilon}_i$ , assumed exogenous and hereafter named *non-wage income*.<sup>21</sup>

From Table 1, we can estimate the migratory flow of workers to the urban sector. When employed rural workers relate their income to the one of the employed urban workers (event 1) and  $y_u - y_r = (\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r) \leq 0$ , then the quantity of immigrant rural workers will be zero. However, when this comparison yields  $y_u - y_r = (\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r) > 0$ , the estimated inflow of employed workers in the urban sector is directly proportional to this differential in income (normalized). In a compact way, the subsequent expression gives both possibilities:

$$(14) \quad (1 - n_u)v_r n_u v_u \max\{(\omega_u + \bar{\epsilon}_u) - (\omega_r - \bar{\epsilon}_r), 0\}.$$

When employed rural workers compare their income with unemployed urban workers (event 2), there will be immigration to the urban sector provided the non-wage income at the urban sector is high enough as weighed against the expected total income in the rural sector:

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<sup>20</sup> In the theory of evolutionary games, this assumption may have several microeconomic foundations. See the appendix to this paper.

<sup>21</sup> The non-wage income may be generalized to include net benefits of the public budget plus other usual sources of income that are studied in the literature on migration. Thus, even under unemployment in both areas, it may be worth the effort to migrate to the urban sector.

$$(15) \quad (1 - n_u)v_r n_u (1 - v_u) \max\{\bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\}.$$

The same type of comparison is valid for events 3 and 4. Provided  $\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r > 0$ , the proportion of rural workers that effectively immigrate is directly proportional to this income difference:

$$(16) \quad (1 - n_u)(1 - v_r)n_u v_u \max\{\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}.$$

Likewise, an encounter of unemployed rural workers with unemployed urban workers, combined with appropriate differential in incomes will result in an inflow of workers to the urban sector given by

$$(17) \quad (1 - n_u)(1 - v_r)n_u (1 - v_u) \max\{\bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}.$$

By aggregating expressions (14), (15), (16) and (17), we get the expected migratory flow from the rural to the urban sector:

$$(18) \quad (1 - n_u)n_u [v_r v_u \max\{\omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\} + v_r (1 - v_u) \max\{\bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r), 0\} \\ + (1 - v_r)v_u \max\{\omega_u + \bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\} + (1 - v_r)(1 - v_u) \max\{\bar{\varepsilon}_u - \bar{\varepsilon}_r, 0\}].$$

### B. Urban-Rural Migratory Flows

A worker from the urban sector becomes a *potential emigrant* only if he compares his income with the income of a worker from the rural sector and discovers that  $y_r = \omega_r + \bar{\varepsilon}_r > \omega_u + \bar{\varepsilon}_u = y_u$ . As before, four events are possible in this pairwise comparison of incomes, according to the situation of employment of each worker in the urban and the rural sectors.

Take, for example, event 1. As employed urban workers contrast their income with the income of their equals as to the employment situation in the rural area, two results are possible. If  $y_u - y_r = \omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r) \geq 0$ , no employed urban worker leaves his area. However, if  $y_u - y_r = \omega_u + \bar{\varepsilon}_u - (\omega_r + \bar{\varepsilon}_r) < 0$ , the migratory outflow from the urban area will be directly proportional to this difference in income, or:

$$(19) \quad -n_u v_u (1-n_u) v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\}.$$

The other three events are interpretable likewise. Thus, from event 3, the result of the meeting of an employed urban worker with unemployed rural workers may be described by:

$$(20) \quad -n_u v_u (1-n_u)(1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\}$$

Event 2, when unemployed urban workers come across employed workers from the rural sector, results in:

$$(21) \quad -n_u (1-v_u)(1-n_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\}.$$

At last, in event 4, unemployed workers from both areas meet and the corresponding expected migration is given by:

$$(22) \quad -n_u (1-v_u)(1-n_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}$$

By aggregating expressions (19) to (22), we attain the expected migratory outflow from the urban to the rural sector:

$$(23) \quad -n_u (1-n_u) [v_u v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_u (1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + (1-v_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + (1-v_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}].$$

### C. Replicator Dynamics

Given these possibilities of migration for the workers, the change in the proportion of workers located in the urban sector is approximated by the difference between the rural-urban flow and the urban-rural flow, i.e., by the difference between (18) and (23):

$$(24) \quad \dot{n}_u = n_u (1-n_u) [v_r v_u \max\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_r (1-v_u) \max\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} \\ + (1-v_r) v_u \max\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} + (1-v_r)(1-v_u) \max\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + v_u v_r \min\{\omega_u + \bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + v_u (1-v_r) \min\{\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r, 0\} \\ + (1-v_u) v_r \min\{\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r), 0\} + (1-v_u)(1-v_r) \min\{\bar{\epsilon}_u - \bar{\epsilon}_r, 0\}].$$

Given that  $\max\{d, 0\} + \min\{d, 0\} = d$  for any real constant  $d$ , the equation (24) may be simplified to:

$$(24-a) \quad \dot{n}_u = n_u(1 - n_u)[(v_u \omega_u + \bar{\epsilon}_u) - (v_r \omega_r + \bar{\epsilon}_r)].$$

This replicator dynamics formally mirrors the intuition that the urban labor force proportion increases, stays put, or lowers while the average income in this market ( $v_u \omega_u + \bar{\epsilon}_u$ ) is, respectively, higher, equal, or lower than in the rural sector ( $v_r \omega_r + \bar{\epsilon}_r$ ).

Optionally, by simple algebraic handling, we get the replicator dynamics (24-b):

$$(24-b) \quad \dot{n}_u = n_u \{ (v_u \omega_u + \bar{\epsilon}_u) - [n_u (v_u \omega_u + \bar{\epsilon}_u) + (1 - n_u)(v_r \omega_r + \bar{\epsilon}_r)] \}.$$

The expression within brackets is the average real income of the economic system as a whole. Consequently, the supply of labor to the urban sector tends to expand if and only if the expected income in this market is superior to the average real income of the economy.

### III. The Harris-Todaro Condition as a Resultant Property of an Evolutionary Game

We have seen in section 2 that the Harris-Todaro model assumes that prices in the rural sector are perfectly flexible, which implies full employment there. This means that  $v_r(t) = 1$  for all  $t \geq t_0$ , with  $t_0$  an arbitrary initial point in time. Moreover, in the urban sector, for a minimum wage, the employment level is  $n_m^* \leq n_u$ . As a result, the probability that a worker finds a job in the urban sector is inversely related to the size of the population in this sector, i.e.,  $v_u = n_m^* / n_u$ . Given that in the Harris and Todaro model non-wage income ( $\bar{\epsilon}_u = \bar{\epsilon}_r = 0$ ) are not taken into account, we have that  $y_u = \omega_u = \bar{\omega}_m$  and  $y_r = \omega_a$ . Given these assumptions, the replicator dynamics, as expressed by (24-a), becomes:

$$(25) \quad \dot{n}_u = n_u(1 - n_u) \left( \frac{n_m^*}{n_u} \bar{\omega}_m - \omega_a \right).$$

The Harris-Todaro condition, as given by (8), is an equilibrium condition in the replicator dynamics of (25). Therefore, the size of the urban population at the long run equilibrium of Harris and Todaro,  $n_u^*$ , is also an equilibrium value from equation of (25). This equilibrium, from the viewpoint of the evolutionary game theory, is an *equilibrium of mixed strategy*, since rural workers keep moving to the urban sector and vice-versa, although the respective sizes of the urban and rural populations are stable. What happens in this kind of

equilibrium is that there is equality between the flow of rural workers that migrate to the urban sector and the flow of urban workers that migrate to the rural sector. In other words, in this mixed strategy equilibrium there is a macroequilibrium generated by intersector migration flows that counterbalance each other.

Worth pointing out is the fact that the replicator dynamics (25), by itself replaces the postulated mechanism of adjustment of Harris and Todaro, which is based on a sign preserving function, i.e., the replicator dynamics substitutes the differential equation (11). Said differently, the mechanism of labor allocation of Harris and Todaro comes out from the interaction of heterogeneous agents in a milieu of bounded rationality.

Now we have just to study the replicator dynamics of (25). Substituting the left-hand side of (8-a) into (25), we get:

$$(25-a) \quad \dot{n}_u = n_u(1 - n_u)\Phi(n_u),$$

remembering that  $\Phi(n_u) \equiv (n_m^*/n_u)\bar{\omega}_m - g'(1 - n_u)\rho(x_m^*/g(1 - n_u))$ .

**PROPOSITION 2:** *The evolutionary dynamics whose state transition is governed by (25-a) converge to the Harris-Todaro (mixed strategy) equilibrium with urban unemployment for any initial condition such that  $n_u(t_0) \in [n_m^*, 1) \in \mathfrak{R}_{++}$ .*

**PROOF:**

Given that  $n_u(1 - n_u) > 0$  for all  $n_u \in [n_m^*, 1)$ , the behavior of the state variable  $n_u$  depends on  $\Phi(n_u)$ , i.e., on the expected wage differential. Since  $g(\cdot) > 0$ ,  $g''(\cdot) < 0$ ,  $\rho(\cdot) > 0$ , and  $\rho'(\cdot) > 0$  for all  $n_u \in [n_m^*, 1)$ , the impact of a variation in urban population on the expected wage differential is given by:

$$(26) \quad \frac{\partial \Phi}{\partial n_u} = -\frac{n_m^* \bar{\omega}_m}{n_u^2} + g''(\cdot)\rho(\cdot) - \rho'(\cdot)x_m^* \left[ \frac{g'(\cdot)}{g(\cdot)} \right]^2 < 0, \text{ for all } n_u \in [n_m^*, 1).$$

Thus in the interval  $[n_m^*, 1)$ , if  $n_u \in [n_m^*, n_u^*)$ , then  $\Phi(n_u) > 0$  and if  $n_u \in (n_u^*, 1)$ , then  $\Phi(n_u) < 0$ . Hence we deduce that for any initial condition  $n_u(t_0) \in [n_m^*, 1)$  the economic system converges to the Harris-Todaro (mixed strategy) equilibrium with urban unemployment,  $n_u^*$ .

If the urban population is below this equilibrium, i.e.,  $n_u \in [n_m^*, n_u^*]$ , then the urban employment rate is too high, the marginal productivity of rural labor is too low and the terms of trade are biased towards manufactured goods. This situation gives rise to an average real wage in the urban sector greater than the real wage in the rural sector. Such a favorable income differential for the urban sector generates a positive net rural-urban migratory flow. This keeps going until its negative effect on the rate of urban employment and its positive effect on the marginal productivity of rural labor and on the terms of trade equalize average real wage between the two sectors. On the contrary, when the urban population is above the level of the Harris-Todaro mixed strategy equilibrium,  $n_u \in (n_u^*, 1)$ , the urban employment rate is too low, the marginal productivity is too high and the terms of trade are biased towards agricultural goods. Now the average real wage in the urban sector is lower than the real wage in the rural sector. This leads to a reversal in rural-urban migration, constraining the migratory movement and taking the system back to the mixed strategy equilibrium. To put in a nutshell, the Harris-Todaro condition emerges as the outcome of the interaction among heterogeneous workers that are worried à la Keynes with their relative wages in conditions of bounded rationality.

#### **IV. Conclusions**

In this paper, we developed an evolutionary game model, more specifically a replicator dynamics, which formalizes the sector allocation of labor as a process of imitation/learning in a milieu of bounded rationality. The Harris-Todaro condition comes out as a spontaneous upshot of interaction among heterogeneous workers.

In the model we propose here, due to the separation of income in two parts – wage and non-wage income – there is room for studying the role of public policies that influence the non-wage component of income and therefore influence the migratory dynamics of Harris and Todaro. Analyzing this dynamics is directly relevant to a better understanding of the process of urban concentration.

APPENDIX:

AN ALTERNATIVE DERIVATION OF THE REPLICATOR DYNAMICS (24-A):

The evolutionary game models based on an *imitation process* are formed by two essential pieces as highlighted by Weibull (1995, p. 152): "the *time rate* at which agents review their strategy choice" and "the *choice probabilities* of a reviewing agent." Let  $x_i$  be the proportion of individuals from the population that opt for the strategy  $s_i$  (*type i agents*), among  $h$  possible strategies and  $x = (x_1, x_2, \dots, x_i, \dots, x_h)$  be the distribution of strategies in the population. The (average) rate of revision of a type  $i$  agent refers to the average number of times this agent revises his strategy per time interval, and may be taken as function of the state of the population,  $r_i(x)$ . This rate of revision may be considered as an *arrival rate* of a Poisson process, i.e., a realization of Poisson process. Let us assume that the frequency with which an agent reconsiders his strategy in a given time interval does not affect the frequency of revisions of the other agents, which is the same as saying that the Poisson process of individual revisions are statistically independent. We then conclude that the aggregate rate of revision of the subpopulation given by type  $i$  agents is by itself a Poisson process with an average rate of revision  $x_i r_i(x)$ . If changes in individual strategies are statistically independent random variables, then the flow of agents from the type  $i$  subpopulation to the type  $j$  subpopulation is an aggregate Poisson process with the arrival rate  $x_i r_i(x) p_i^j(x)$ , where  $p_i^j(x)$  is the probability of a type  $i$  agent becoming a type  $j$ . The choice probabilities of a type  $i$  agent form a probability distribution, represented by  $p_i(x) = (p_i^1(x), \dots, p_i^j(x), \dots, p_i^h(x))$ , naturally with  $\sum_{j=1}^h p_i^j(x) = 1$ .

In the case of only two possible strategies,  $h = 2$ , we have  $x_1 + x_2 = 1$ . Thus we need to consider only one state variable. Without loss of generality, we will take  $x_1$  as the reference state variable, so the vector of distribution of strategies will be perfectly determined once  $x_1$  is given, because  $x = (x_1, x_2) = (x_1, 1 - x_1)$ . The inflow to the subpopulation of type  $i$  agents can be approximated by

$$(27) \quad x_2 r_2(x) p_2^1(x)$$

and the outflow by

$$(28) \quad x_1 r_1(x) p_1^2(x).$$

The rate of variation of subpopulation 1 will be determined by the difference between inflow and outflow:

$$(29) \quad \dot{x}_1 = x_2 r_2(x) p_2^1(x) - x_1 r_1(x) p_1^2(x).$$

As explained by Weibull (1995, sect. 4), there are several assumptions pertaining to the revision rate and to the probability of choice that may be adopted. In section 3 above, we implicitly assumed that all reviewing agents reassess their choice together and just once per time interval, or

$$(30) \quad r_1(x) = r_2(x) = 1$$

As in section 3, we assume a *direct imitation process* (Weibull, 1995, p. 155-158), i.e., each reviewing agent randomly meets another agent so that the probability of a reviewing agent encountering an agent that follows strategy  $s_1$  is  $x_1$  and one that follows  $s_2$  is  $x_2 = 1 - x_1$ . As done by Weibull (1995, p. 155-158), we suppose that, after the reviewing agent meets another agent, he detects, with some noise, his own payoff,<sup>22</sup>  $\pi_i(x) + \alpha_i$ , and the payoff linked to the strategy of the other agent,  $\pi_j(x) + \alpha_j$ . Since  $\alpha_i$  and  $\alpha_j$  are stochastic variables, the difference  $\alpha_i - \alpha_j$  is also a stochastic variable, whose cumulative distribution function  $F : R \rightarrow [0,1]$  is assumed to be continuously differentiable.

The type  $i$  reviewing agent becomes a type  $j$  agent if and only if the *observed* result of the latter overcomes the *observed* result of the former, i.e.,  $\pi_j(x) + \alpha_j > \pi_i(x) + \alpha_i$ , or equivalently,  $\alpha_i - \alpha_j < \pi_j(x) - \pi_i(x)$ . The probability that this happens is given by the image of the cumulative distribution function at the point  $\pi_j(x) - \pi_i(x)$ , that is,  $F(\pi_j(x) - \pi_i(x))$ . This function may be interpreted in other manners. Weibull (1995, p. 157), for example, gives an alternative rendition to the variables  $\alpha_i$  and  $\alpha_j$ . They may be seen as idiosyncratic differences between preferences of the agents. As a result,  $\pi_i(x) + \alpha_i$  and  $\pi_j(x) + \alpha_j$  become the true payoffs from strategies  $s_i$  and  $s_j$ , and the choice probabilities become the result from the differences between the strategies of agents, instead of individual observational errors by agents with similar preferences. Nachbar (1990) gives an alternative

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<sup>22</sup>  $\pi_i(x)$  is the expected result for an agent that opts for a strategy  $s_i$  when the population state is  $x = (x_1, x_2) = (x_1, 1 - x_1)$ .

interpretation to function  $F(\cdot)$ , in which it reflects a change cost,  $c$ , not given in a deterministic form and instantly paid. In fact,  $F(\cdot)$  is assumed as a cumulative distribution function that furnishes the probability that  $c \leq \pi_j(x) - \pi_i(x)$ . Hence, when a type  $i$  reviewing agent compares his payoff with a type  $j$ , he only changes his strategy if the cost of changing is lower or equal to the difference between the payoffs. In short, these authors include observational errors, distinctions in preferences, or stochastic adjustment costs in order to give a microeconomic foundation to the imitation-based selection dynamics.

Summarizing, the probability that an agent of type  $i$  meets an agent of type  $j$  is  $x_j$ , and the probability that a type  $i$  agent, given that he has met a type  $j$ , choose a strategy  $s_j$ , is  $F(\pi_j(x) - \pi_i(x))$ . Assuming that these two events are statistically independent, then the probability distribution function for the choice of a type  $i$  agent may be so expressed:

$$(31) \quad p_i(x) = (p_i^j(x), p_i^i(x)) = (x_j F(\pi_j(x) - \pi_i(x)), 1 - x_j F(\pi_j(x) - \pi_i(x))),$$

for  $j \neq i$  and  $i, j = 1, 2$ . Substituting (30) and (31) into (29), we get:

$$(32) \quad \dot{x}_1 = x_1(1 - x_1)[F(\pi_1(x) - \pi_2(x)) - F(\pi_2(x) - \pi_1(x))].$$

Now suppose that the stochastic variable  $\alpha_i - \alpha_j$  be uniformly distributed, i.e., that its probability density function be:

$$(33) \quad f(\alpha_i - \alpha_j) = \begin{cases} \beta, & -a \leq \alpha_i - \alpha_j \leq a \\ 0, & \text{elsewhere} \end{cases},$$

with support  $[-a, a]$ , in which  $a = \max\{|\pi_1(x) - \pi_2(x)|, \forall x \in \Delta^2\}$ , with  $\beta > 0$  being a real constant. We then get the following cumulative distribution function:

$$(34) \quad F(\pi_j(x) - \pi_i(x)) = \frac{[\pi_j(x) - \pi_i(x)] + a}{2a}, \text{ with } j \neq i.$$

Inserting (34) into (32), we attain:

$$(35) \quad \dot{x}_1 = (1/a)x_1(1 - x_1)[\pi_1(x) - \pi_2(x)].$$

In the replicator dynamics (24-a), from section 3, we have that  $x_1 = n_u$  and  $x_2 = n_r = 1 - n_u$ , thus the type 1 agent is the urban worker and the type 2 agent is the rural worker. The expected payoff of a type 1 agent (urban worker) becomes:

$$(36) \quad \pi_1(x) = v_u \omega_u + \bar{\epsilon}_u,$$

and the respective expected payoff of a rural worker turns into:

$$(37) \quad \pi_2(x) = v_r \omega_r + \bar{\epsilon}_r.$$

For this reason, by considering (36) and (37), the replicator dynamics may be written as:

$$(38) \quad \dot{n}_u = (1/a)n_u(1-n_u)[(v_u \omega_u + \bar{\epsilon}_u) - (v_r \omega_r + \bar{\epsilon}_r)],$$

in which

$$a = \max\{|\pi_1(x) - \pi_2(x)|, \forall x \in \Delta^2\} = \max\{|v_u \omega_u + \bar{\epsilon}_u - (v_r \omega_r + \bar{\epsilon}_r)|, \forall n_u \in (0,1) \in \mathfrak{R}\}.$$

Equation (38) is the replicator dynamics (24-a), except for the scalar  $1/a$ , which does not alter the qualitative structure of the solutions of the replicator dynamics under inquiry. As a result, (24-a) may be interpreted as a normalized version of (38). Normalizing  $a = 1$ , we can say that the income difference in (24-a) may now be expressed in units of the maximum differential in average income that is a priori possible.

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Table 1 – Income differential and number of estimated comparisons between workers of urban and rural sectors, according to employment situation

		Event			
		1	2	3	4
Urban sector worker	Situation	Employed	Unemployed	Employed	Unemployed
	Income ( $y_u$ )	$\omega_u + \bar{\epsilon}_u$	$\bar{\epsilon}_u$	$\omega_u + \bar{\epsilon}_u$	$\bar{\epsilon}_u$
	Sub-population	$n_u v_u$	$n_u (1 - v_u)$	$n_u v_u$	$n_u (1 - v_u)$
Rural sector worker	Situation	Employed	Employed	Unemployed	Unemployed
	Income ( $y_r$ )	$\omega_r + \bar{\epsilon}_r$	$\omega_r + \bar{\epsilon}_r$	$\bar{\epsilon}_r$	$\bar{\epsilon}_r$
	Sub-population	$(1 - n_u)v_r$	$(1 - n_u)v_r$	$(1 - n_u)(1 - v_r)$	$(1 - n_u)(1 - v_r)$
Income differential ( $y_u - y_r$ )		$(\omega_u + \bar{\epsilon}_u) - (\omega_r + \bar{\epsilon}_r)$	$\bar{\epsilon}_u - (\omega_r + \bar{\epsilon}_r)$	$\omega_u + \bar{\epsilon}_u - \bar{\epsilon}_r$	$\bar{\epsilon}_u - \bar{\epsilon}_r$
Number of estimated comparisons		$n_u v_u (1 - n_u)v_r$	$n_u (1 - v_u)(1 - n_u)v_r$	$n_u v_u (1 - n_u)(1 - v_r)$	$n_u (1 - v_u)(1 - n_u)(1 - v_r)$

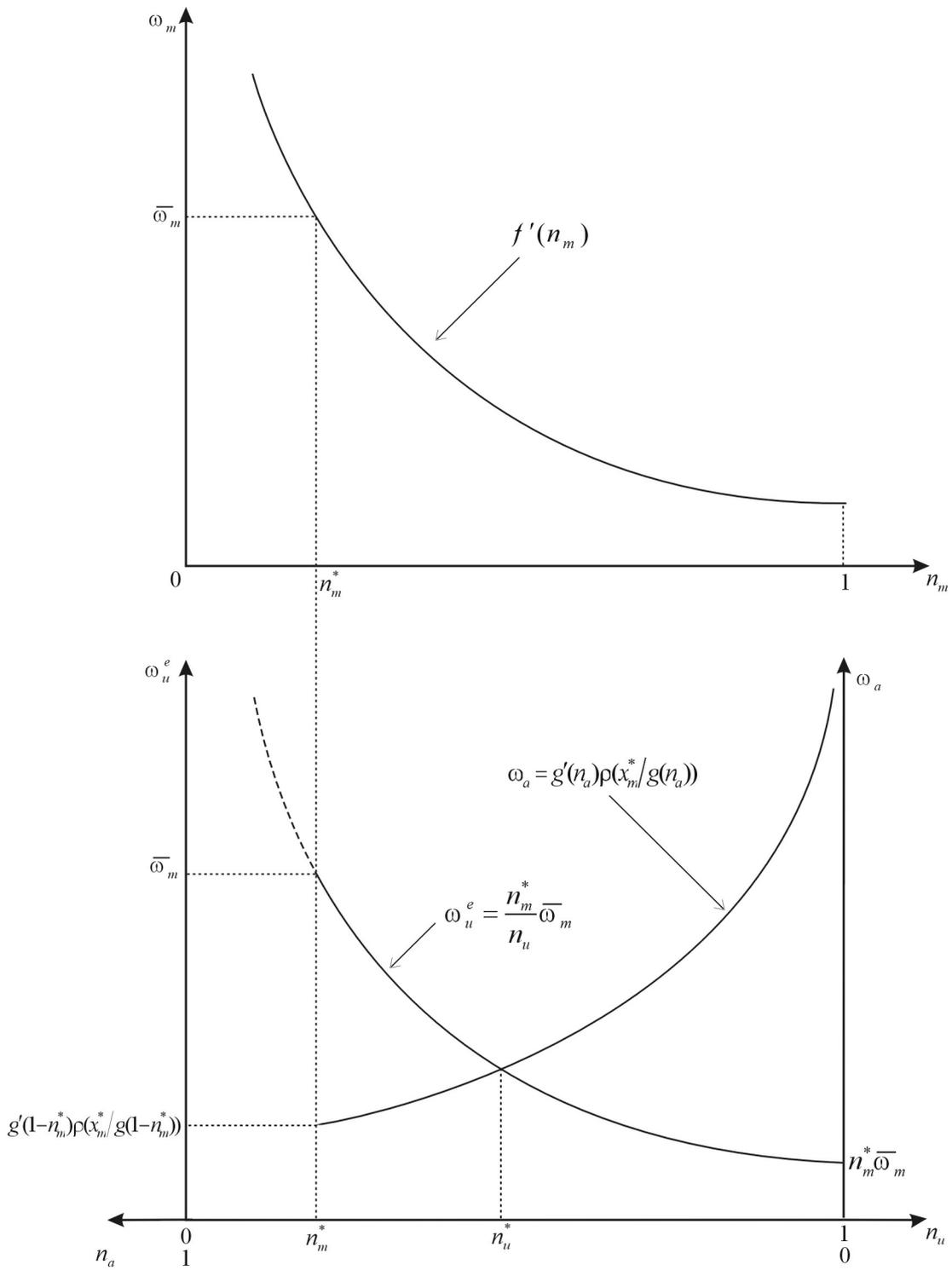


Figure 1 – Harris-Todaro model with endogenous terms of trade